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Technical Note

Transient analysis of internally heated tubular components with exponential thermal loading and external convection

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Nomenclature

a inner radius
b outer radius
Bi Biot number, bh/k
c exponential term
C_0 equation (2.1)
C_1 equation (2.2)
C_2 equation (2.3)
Fo Fourier number, $\kappa t/b^2$
h convection coefficient
J_i, Y_i Bessel functions of ith order
k thermal conductivity
r radius, $a \leqslant r \leqslant b$
t time
$T(r, t)$ temperature at radius r and at time t
U step temperature change
V final temperature.

Greek symbols
β_n roots of characteristic equation
κ thermal diffusivity
$\phi(r, t)$ unit step response.

1. Introduction

Tubular components subjected to severe thermal conditions are widely used by industry in many applications such as radiant burners and heat exchangers. As would be expected, these applications often involve thermal transients that are severe enough to induce fatigue, and eventually failure. In order to be able to predict these failures and the thermoelastic stresses that are the underlying cause, a detailed understanding of the transient temperature distributions is essential. Although there have been numerous analytical models, they have been limited to severe and often unrealistic step or linear temperature changes [1, 2]. More recent studies [3–5] have shown the potentially complicated time dependence of the surface temperature loading and need for improved estimates using finite-element analysis along with a temperature-matching scheme when temperature dependent materials properties are involved. However, the finite element calculations were iterative and required an analytical starting point for the prescribed surface temperatures, as well as a means for verifying the numerical solution. Because of these needs, an analytical model of the thermal transients developed within tubular components subjected to a more realistic, time dependent boundary condition is ultimately required. Accordingly, this paper derives the equations for a hollow cylinder with a plausible exponential boundary condition of the form $H(t) = V(1-e^{-ct})$ for the internal surface with external convection to the external environment. Additionally, the unit response of a cylinder subjected to an internal step load with external convection is also derived.

2. Analytical considerations

A number of years ago a general purpose series solution was derived [6] for an infinite hollow circular cylinder

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subjected to mixed boundary conditions involving step temperature changes or convection on the internal and external surfaces. Using this generalized solution, the temperature change across the radius of the cylinder due to an internal, unit step increase with external convection can be shown to be:

$$\phi(r,t) = U\frac{1-Bi\ln(r/b)}{1-Bi\ln(a/b)} + \pi\sum_{n=1}^{\infty} e^{-\kappa\beta_n^2 t}\frac{C_0(r,\beta_n)C_1^2(b,\beta_n)}{C_2(a,b,\beta_n)} \quad (1)$$

where U is the magnitude of the internal step temperature change, r is the radius at the point of evaluation, a is the internal radius, b is the external radius, κ is the material's thermal diffusivity (assumed to be independent of temperature), t is the time, and the functions C_0, C_1, and C_2 are defined as:

$$C_0(r,\beta_n) = J_0(r\beta_n)Y_0(a\beta_n) - Y_0(r\beta_n)J_0(a\beta_n) \quad (2.1)$$

$$C_1(b,\beta_n) = k\beta_n J_1(b\beta_n) - hJ_0(b\beta_n) \quad (2.2)$$

$$C_2(a,b,\beta_n) = (k^2\beta_n^2 + h^2)[J_0(a\beta_n)]^2 - [k\beta_n J_1(b\beta_n) - hJ_0(b\beta_n)]^2. \quad (2.3)$$

In the preceding equations, β_n represents the real and simple roots of the characteristic equation:

$$J_0(a\beta)\left[\beta Y_1(b\beta) - \frac{h}{k}Y_0(b\beta)\right] - Y_0(a\beta)\left[\beta J_1(b,\beta) - \frac{h}{k}J_0(b,\beta)\right] = 0. \quad (3)$$

Equation (1) is a versatile relationship that can be used to conservatively calculate the transient response of a cylinder to a very severe thermal event. However, most thermal shock events will not be instantaneous and will exhibit some time dependence of the surface temperature load.

For a more plausible time dependent boundary condition described by a decaying exponential such as:

$$H(t) = V(1-e^{-ct}) \quad (4)$$

the response can be determined under the assumption of constant thermophysical properties by using the following form of Duhamel's integral [6]

$$T(r,t) = \int_0^t \frac{\partial H(\tau)}{\partial \tau}\phi(r,t-\tau)\,\mathrm{d}\tau \quad (5)$$

provided $H(0) = 0$ and the initial temperature is zero. Substituting equation (1) and the derivative of equation (4) into equation (5) and integrating with respect to time, the expression for temperature as a function of radius and time for an internal, exponential boundary condition can finally be derived as:

$$\frac{T(r,t)}{V} = (1-e^{-ct})\frac{1-Bi\ln(r/b)}{1-Bi\ln(a/b)} + \pi c e^{-ct}\sum_{n=1}^{\infty}\left[\frac{1-e^{-(\beta_n^2\kappa - c)t}}{(\beta_n^2\kappa - c)}\right]\frac{C_0(r,\beta_n)C_1^2(b,\beta_n)}{C_2(a,b,\beta_n)} \quad (6)$$

provided $\beta_n^2\kappa - c \neq 0$ to avoid the singularity and U (not shown) is set to unity.

The first term in equation (1) represents the stationary distribution which would be established if the prescribed temperature $H(t)$ would be fixed at time t, whereas the second term accounts for the lag in the temperature distribution behind the stationary. As would be expected, the second term eventually vanishes and the solution approaches the expected logarithmic, steady state distribution at large values of time.

3. Discussion

In order to verify the accuracy and versatility of the derived relationships, a series of thermal transient calculations using finite-element analysis (FEA) were performed. All confirmatory FEA calculations were performed using the ANSYS code with a single row of 20 axisymmetric elements with axially coupled nodes to simulate an infinite cylinder. In addition, all material properties were assumed to be independent of temperature. For the step temperature change calculations, the internal surface temperature was instantaneously applied to the nodes on the inner surface of the model. In contrast, the exponential boundary conditions were imposed on the inner nodes in a piece-wise linear fashion using 1 s time increments. For both calculations, a convective coefficient assumed to be independent of temperature was uniformly applied to the outer element face.

Initially, the step response to a cylinder was analytically and numerically modeled using the following dimensions and thermophysical properties: $a = 4$ cm, $b = 6$ cm, $k = 0.1$ W cm^{-1} °C^{-1}, $h = 0.2$ W cm^{-2} °C^{-1} and $\kappa = 0.83$ cm^{-2} s^{-1}. As can be seen in Fig. 1, excellent agreement was obtained between the analytical solution using the first 100 terms of the series and finite element predictions over a wide range of time. The response to an exponential thermal loading on the internal surface in the form of $H(t) = 100(1-e^{-0.5t})$ was then evaluated using equation (6) as shown in Fig. 2. Again, excellent agreement can be seen between the analytical and numerical solutions over a wide range of time up to steady-state.

Although the mathematics involved with the derivations are straightforward, equations (1)–(6) still represent versatile, and hitherto underived tools that can be used to calculate the transient response of an internally loaded cylinder with external convection. Because of their versatility, the derived relationships can be used during the design and analysis process, as well as to augment

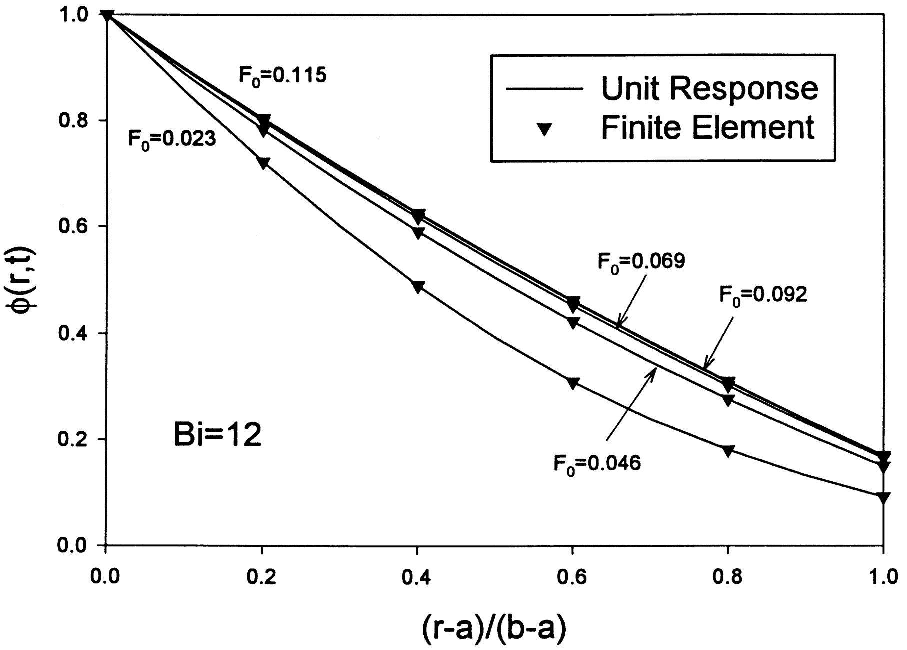

Fig. 1. Transient temperature distribution across the radius of a cylinder subjected to a step temperature change on the internal surface and convection on the outer surface.

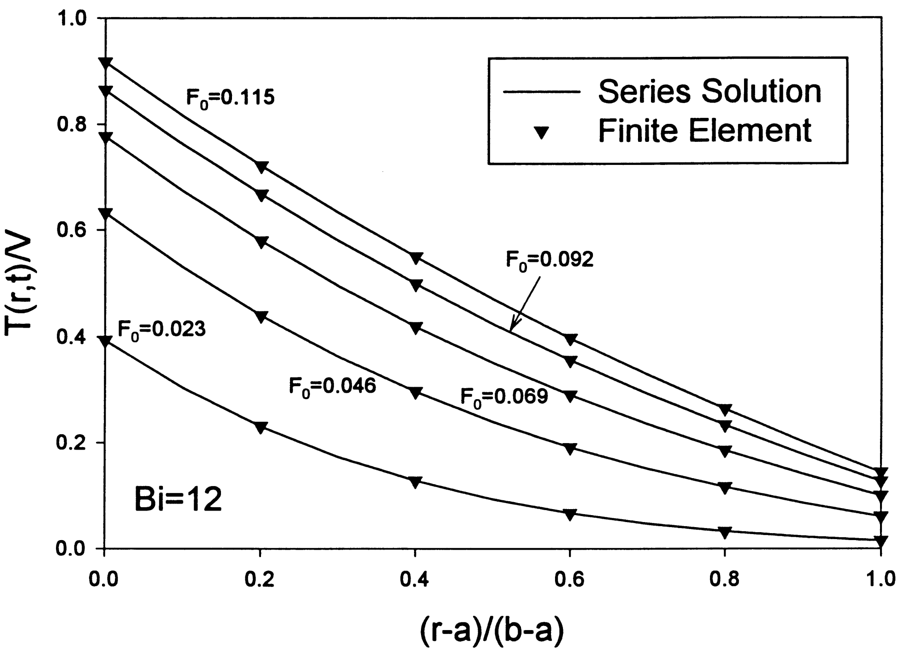

Fig. 2. Transient temperature distribution across the radius of a cylinder subjected to exponential heating on the internal surface and convection on the outer surface.

and/or verify iterative FEA calculations when limited temperature data exists. In either case, the derived relationships offer a practical and fast means for investigating the influence of time dependant boundary conditions, thermal properties, and geometry.

4. Conclusions

Using Duhamel's integral and the unit step response derived for a hollow cylinder, an analytical solution has been derived for the transient temperature distribution across a tubular component internally subjected to exponential thermal loading with external convection to the surrounding environment. Excellent agreement was seen between the derived solution and a piece-wise linear, finite-element simulation. Because of the flexibility of the exponential boundary condition used for the analysis, the derived, closed-form solution has many practical research and industrial applications including the design process, as well as providing a first guess for an iterative, finite element analysis when materials nonlinearities are involved.

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